

SEQUENCES

Triangular numbers

A triangular number is a natural number such that the shape of an equilateral triangle can be formed by that number of points.



For $n \geq 1$, let T_n be the n^{th} triangular number. By convention $T_0 = 0$.

1. Find the six first triangular numbers.
2. For $n \geq 1$, let U_n be the difference between the n^{th} triangular number and the previous one. That is to say $U_n = T_n - T_{n-1}$. Give the first six terms of the sequence (U_n) . What kind of sequence could it be ?
3. Using the previous result, try to find out the recurrent relation between T_n and T_{n-1} with n going from 1 to 6. Now add, member by member, these six relations. Finally write T_6 as a sum of the different numbers.
4. Deduce a formula to find directly the n^{th} triangular number
5. Use this formula to calculate the 36^{th} triangular number. Is this number famous for other reasons?

Vocabulary: shape: forme (d'une figure géométrique)