

Sujet n°28

Please, do not write on this exam paper and don't forget to give it back at the end of the test.

SEQUENCES

The Sierpinski triangle :

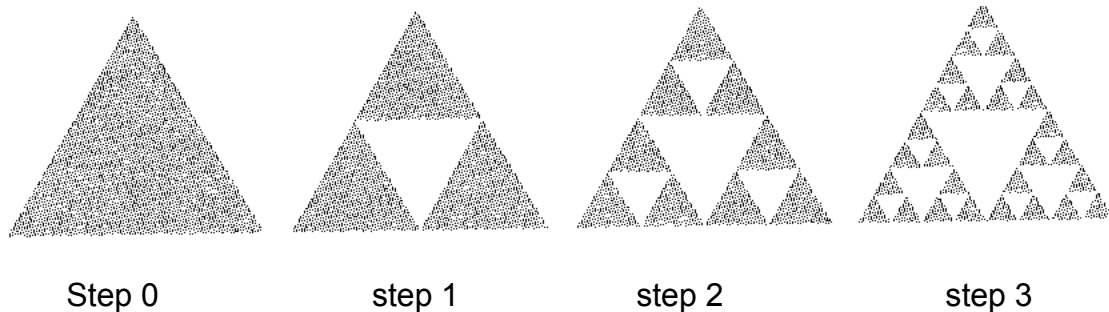
1) Let S_n be a series defined for all n in \mathbb{N} by : $S_n = 1 + 3 + 3^2 + 3^3 + 3^4 + \dots + 3^{n-1}$.

Work out S_1 , S_2 , S_3 and S_{10} .

2) The Sierpinski triangle is a fractal named after the Polish mathematician Waclaw Sierpinski who described it in 1915.

Drawing :

We start with one grey equilateral triangle, at first step we divide this triangle in four similar sub-triangles, and replace the triangle in the middle by a white triangle. Then, at every following step, we iterate this method on each remaining grey triangle .



a) How many white triangles are there at step 10 ? Justify your answer.

b) Calculate $\lim_{n \rightarrow +\infty} \frac{1 - (\frac{3}{4})^n}{1 - \frac{3}{4}}$.

c) Let's assume that the ratio of the white area and the first grey triangle area is given,

at step n , by the formula : $r_n = \frac{1}{4} + \frac{3}{4^2} \left(\frac{1 - (\frac{3}{4})^n}{1 - (\frac{3}{4})} \right)$

Deduce which ratio of the first grey triangle, the white area represents when the number of steps tends to infinity.