Sujet n°7

SEQUENCES

Please do not write on the subject paper and don't forget to give back the examination paper at the end of the test.

<u>Indroduction :</u>
There are many mathematical and recreational problems related to folding. Origami, the Japanese art of paper folding, is one well-known example.
It is possible to make a surprising variety of shapes by folding a piece of paper multiple times, making one complete straight cut, then unfolding. For example, a five-pointed star can be produced after four folds, as can a polygonal swan, butterfly, and angelfish.
In January 2002, while a junior in high school, Britney Gallivan demonstrated that a single piece of toilet paper 4000 ft (1200 m) in length can be folded in half eleven times. This was contrary to the popular conception that the maximum number of times any piece of paper could be folded in half was seven.
Britney and her feat were mentioned in the Season 1 episode "Identity Crisis" (2005) of the television crime drama NUMB3RS.

Adapted from http://mathworld.wolfram.com/Folding.html

<u>Questions :</u>
A piece of paper is 0.04 inch thick. By folding into half, the thickness becomes 0.08 inch and so on. Imagine it can be folded as many times as desired.

1. We modelise this situation by a sequence (u_n) so that $u_0 = 0.04$
   a) What sort of progression is it ?
   b) Give the recursive formula. (u_{n+1} in terms of u_n)
   c) Express u_n in terms of n.

2. What is the thickness of the paper after it has been folded 5 times ?

3. What is the thickness of the paper after it has been folded 20 times ? Convert into feet knowing that a foot is equal to 12 inches.

4. How many times should the paper be folded for its thickness to be 750 inches or more ? (750 inches or 3 m is about the height of a room)

5. How many times should the paper be folded for its thickness to reach the moon (the average distance between the Earth and the Moon is 238,857 miles) ? One mile equals 5,280 feet.