Subject 02   **ARITHMETIC**

Please do not write on this exam paper and give it back at the end of the test

ARITHMETIC: PRIME NUMBERS AND GREATEST COMMON DIVISOR

1 : General question
Do you know what a prime number is? Can you give a list of several primes? Do you know how many primes are there? In which fields are primes very useful?

2 : Prime Factorization
It is often useful to write a number in terms of its prime factorization, or as the product of its prime factors. For example, 56 can be written as 2×2×2×7 and 84 can be written as 2×2×3×7 . Every number can be written as a product of primes, and, like a fingerprint, every number has a unique prime factorization.

To write down a prime factorization of a number, start by dividing the number by its lowest prime factor. Write down this factor, and divide the *new number* by its lowest prime factor (it does not matter if this is the same as the first prime factor). Write this factor down and divide the new number by its lowest factor. Continue in this manner until the resulting number is prime. Write this number down as the final factor.

Example : Compute the prime factorization of 1,575.
Step 1. Is 1,575 divisible by 2? No. By 3? Yes. 1, 575/3 = 525 . Write down 3.
Step 2. Is 525 divisible by 3? Yes. 525/3 = 175 . Write down 3.
Step 3. Is 175 divisible by 3? No. By 5? Yes. 175/5 = 35 . Write down 5.
Step 4. Is 35 divisible by 5? Yes. 35/5 = 7 . Write down 5.
Step 5. 7 is prime. Write down 7.
Therefore, the prime factorization of 1,575 is 3×3×5×5×7 .

Exercise: Compute the prime factorization of 23,100.

2 : Greatest Common Factor

A common factor of two numbers is a factor that divides both numbers. The greatest common factor (GCF) of two numbers is the greatest number that divides both numbers. To find the GCF, take the prime factorization of both numbers. Then write down the factors that they have in common. If they share more than one of the same factor (two 2's, for example), write them both down. Then multiply the factors they have in common.
For example, the greatest common factor of 1,575 and 23,100 is 3×5×5×7 = 525 . 1,575 and 23,100 are both divisible by 525, and they are not both divisible by any number greater than 525.

3 : Relative Primes

Sometimes, two numbers do not have any prime factors in common. For example, the prime factorization of 40 is 2×2×2×5 and the prime factorization of 21 is 3×7 . Since 40 and 21 have no common prime factors, they are said to be relatively prime, and their greatest common factor is 1.

Exercise: Compute the GCD of 1,575 and 23,100 .
Are they relatively primes? How can a prime factorization be used to compute the GCD of 2 numbers?