

Subject n°26

functions

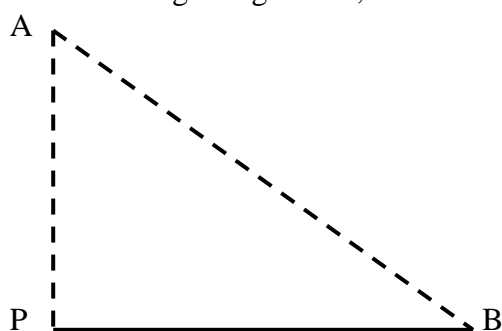
Please do not write on the exam paper, and do not forget to give back the examination paper at the end of the test.

Orienteering is a family of sports that requires navigational skills using a map and compass to navigate from point to point in diverse and usually unfamiliar terrain, and normally moving at speed. Participants are given a topographical map, usually a specially prepared orienteering map, which they use to find control points. Originally a training exercise in land navigation for military officers, orienteering has developed many variations.
source: <http://en.wikipedia.org/wiki/Orienteering>

Three different teams enter a foot orienteering competition.

The two first control points, A and B, are separated as shown in the following figure:

APB has a right angle at P, $AP = PB = 2\text{km}$.



The path from P to B is well-trodden, therefore runners can run fairly quickly along it, at an average speed of 10km/h.

But the rest of the grounds is far rougher, and can only be crossed at an average speed of 6km/h.

1) Because of the layout of the grounds, the first team plans to go straight from A to P, then from P to B, so as to spend as little time as possible on rough grounds.
Calculate the total time this first team will need (in min).

2) Since geometrically wise the straight line is the shorter route between two points, the second team decides to aim straight for B across the rougher grounds.
Calculate the total time the second team will need (in min and s, rounded to the nearest s).

3) The third team has a calculus genius among its members. His or her strategy is as follows:
3-a) Let us say we aim for a point Q, situated on the segment [PB] at x km from P.
We consider now the time function T, giving the total time needed to go from A to B in terms

of x: prove that $T(x) = \frac{\sqrt{4+x^2}}{6} + \frac{2-x}{10}$

3-b) Show that, if we want to minimize our time, x may be a solution to the equation

$$10x = 6\sqrt{4+x^2}$$

3-c) Find the value of x.

Let us assume this value actually minimizes the time.

3-d) Calculate the time we will need (in min).

Who will win?

NB: "well-trodden" means "très fréquenté"