

### Subject 9

**Please do not write on the exam paper.**

We suppose that, for a certain period of time, a country's population is **constant** and equals **40 million** inhabitants, among whom 10 million live in towns and 30 million live in rural zones.

We observe that the population movements are described by the following rule :  
each year, 10 % of the town persons emigrate to rural zones and 30 % of the rural persons emigrate to towns.

We respectively note  $T_n$  and  $R_n$  the number of persons (in million) who live in towns and in rural zones after  $n$  years ( $T_0 = 10$  and  $R_0 = 30$ ).

**a)** Prove that, for all whole number  $n$ , we have :

$$T_{n+1} = 0.9T_n + 0.3R_n \quad \text{and} \quad R_{n+1} = 0.1T_n + 0.7R_n .$$

**b)** Prove that  $T_n + R_n$  is a constant.

**c)** Deduce from **b)** that the sequences  $(T_n)$  and  $(R_n)$  satisfy to :

$$T_{n+1} = 0.6T_n + 12 \quad \text{and} \quad R_{n+1} = 0.6R_n + 4 .$$

**d)** The sequences  $(T_n)$  and  $(R_n)$  are not geometrical, but prove that  $(t_n)$  and  $(r_n)$ , defined by (  $t_n = T_n - 30$  and  $r_n = R_n - 10$  ) are geometrical sequences.

**e)** Express  $T_n$  and  $R_n$  in terms of  $n$ .

Then study the limits of the sequences  $(T_n)$  and  $(R_n)$ . What can you conclude ?