

SUBJECT 4

Please, do not write on the exam paper

When the creator of the game of chess showed his invention to the king of the country, the king was so pleased that he gave the inventor the right to choose his reward. The man asked the king this: for the first square of the chess board, he would receive one *wheat seed*, two for the second one, four for the third one, and so forth, doubling the amount each time. The king, arithmetically illiterate, thought the price was low and accepted the inventor's offer. He ordered the treasurer to count and give the wheat to the inventor. But the treasurer explained that it would be impossible to give the inventor the reward.

We denote by u_n the number of wheat seeds on the n^{th} square. So $u_1 = 1$.

1. Compute u_2 and u_3 .
2. Express u_n in terms of n .
3. Find out the total number of wheat seeds on the chess-board.
4. Given that the average weight of a seed of wheat is 40 milligrams, calculate how many tons should have been offered as a reward to the inventor.
5. In 2011, the world production of wheat reached 650 millions of tons. Did the inventor really ask for such a low price?

Nota:

- *wheat seed* : grain de blé.

- For $q \neq 1$, $1 + q + q^2 + \dots + q^n = \frac{1 - q^{n+1}}{1 - q}$