

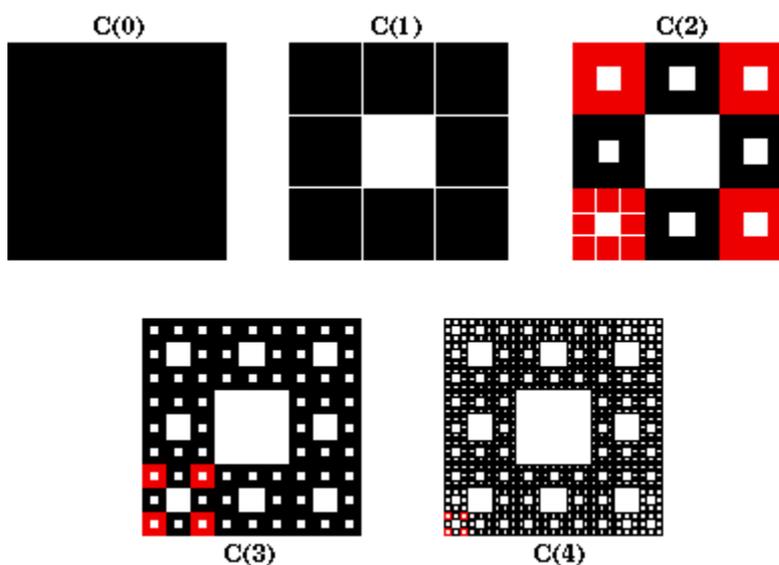
## Subject 19

**Please, don't write on the exam paper.**

### The Sierpinski Carpet

The construction of the Sierpinski carpet begins with a square of side 1. The square is cut into 9 identical subsquares in a 3-by-3 grid, and the central subsquare is removed. The same procedure is then applied recursively to the remaining 8 subsquares.

The Sierpinski carpet at order  $n$  is denoted by  $C(n)$ . We denote by  $A_n$  the area of  $C(n)$ .



- 1) Using the preceding image and description, explain how we obtain  $C(2)$ ,  $C(3)$  and  $C(4)$ .
- 2) Compute the areas  $A_0$ ,  $A_1$ ,  $A_2$  of  $C(0)$ ,  $C(1)$ ,  $C(2)$  respectively.
- 3) Justify that :  $A_{n+1} = \frac{8}{9}A_n$ .
- 4) What kind of sequence is  $(A_n)$ ? Write  $A_n$  in terms of  $n$ .
- 5) Is  $(A_n)$  an increasing or decreasing sequence?
- 6) What is the proportion of  $C(n)$  vanishing at each step?
- 7) What is the smallest  $n_0$  for which we have  $A_{n_0} < 0,1 A_0$  ?

### Optional questions

- 8) Justify that  $C(n+1)$  is included in  $C(n)$ .
- 9) Justify that  $C(n) = C(0) \cap C(1) \cap \dots \cap C(n)$  (use a recurrency if you are in S section, otherwise state it for  $n=1,2,3$ )
- 10) We denote by  $C = \bigcap_{n \geq 0} C(n)$  the intersection of all  $C(n)$ . This is the limit set  $C$  of  $C(n)$  as  $n$  goes to infinity, called the Sierpinski Carpet. Make a conjecture concerning its area. Is  $C$  an empty set ?