

Subject 12

Function and sequence

Please, don't write on the exam paper.

Let f be a function defined on $[0;5]$.

We assume that f is increasing over $[0;5]$ and that $f(0) = 1$ and $f(5) = \frac{7}{2}$

Let (u_n) be the sequence defined by $u_{n+1} = f(u_n)$ for every integer n .

- 1) **a)** Sketch the graph of a possible function f .
 b) Represent some values of the sequence on the graph.

2) We assume here that $u_0 = 0$.

- a)** Prove that for every integer n , $u_n \in [0;5]$.
b) Prove that for every integer n , $u_n \leq u_{n+1}$.
c) What can be deduced for the sequence (u_n) ?

3) We assume here that $u_0 = 5$.

- a)** Prove that for every integer n , $u_n \in [0;5]$.
b) Prove that for every integer n , $u_n \geq u_{n+1}$.
c) What can be deduced for the sequence (u_n) ?