

**Subject 1**  
**Sequences**

Please, do not write on the exam paper.



**The Cantor set**

The Cantor set is created by repeatedly deleting the open middle thirds of a set of line segments. One starts by deleting the open middle third  $]1/3, 2/3[$  from the interval  $[0, 1]$ , leaving two line segments. Next, the open middle third of each of these remaining segments is deleted, leaving four line segments...

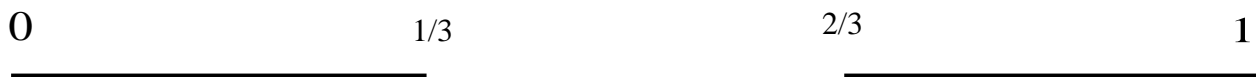
**A) Constructing the Cantor set**

Begin with set  $C_0 = [0; 1]$



1) What is the length  $l_0$  of  $C_0$  ?

2) Now remove the open middle third of this interval,  $]1/3, 2/3[$ , leaving two closed intervals behind.



This will be the set  $C_1$ .

a) Write  $C_1$  in interval notation. What is the length  $l_1$  of  $C_1$  ?

b) The complementary set of  $C_1$  (in  $C_0$ ) is denoted by  $C_1'$ . What is the length of  $C_1'$  ?

3) Same questions for  $C_2$  :



## B) Optional questions

**If the subject is easy for you and if you still have time to do it, continue with these optionnal questions :**

$C_n$  being the  $n^{th}$  set constructed with this process, we obtain the  $(n+1)^{th}$  set  $C_{n+1}$  by removing the middle third of each segment contained in  $C_n$ .

We call  $l_n$  the length of  $C_n$ . We want to calculate  $l_n$ .

1) Explain why  $l_{n+1} = \frac{2}{3}l_n$ . What kind of sequence is  $(l_n)$ ?

2) Compute  $l_n$  in terms of  $n$ .

3) Compute the limit of  $l_n$  as  $n$  tends to infinity.

4) We call  $C$  the intersection of all sets  $C_n$ . This is the final Cantor set. What is its length? Is it an empty set?