

Subject n°22

Please don't write on the exam paper.

Let (f_n) be the Fibonacci sequence.

$$f_0 = 0, f_1 = 1, f_{n+1} = f_n + f_{n-1} \quad (1) \text{ for all } n \geq 1.$$

1) Compute the first 10 terms of the sequence.

2) Let $\Phi_n = \frac{f_{n+1}}{f_n}$. Dividing by f_n in the recurrence equation (1), prove that Φ_n satisfies the equation : $\Phi_n = 1 + \frac{1}{\Phi_{n-1}}$.

3) We suppose that Φ_n has a limit called Φ .

Prove that this limit is positive and satisfies the equation : $\Phi = 1 + \frac{1}{\Phi}$.

4) Find Φ .

5) Explain how the following picture can be related to the Fibonacci sequence.



